Czech University of Life Sciences Prague
Faculty of Environmental Sciences

Calculation of drainage retention capacity R(t)

27.10.2012

Calculation of drainage retention capacity R(t)

Introduction

For expression of the drainage retention capacity R(t) (M) as a function of time was used the unsteady-state approach. This method comes out from the description of lowering of the ground water table in time, in the unsteady-state saturated flow conditions, in a case of wetted soil surface, where the initial water table level h_0 (M) at time t = 0 is identical with drain depth h_d (M). Symbol M represents unit of length.

Unsteady-state drainage equations are based on the differential equations for unsteady groundwater flow. The base of those methods and processes can be applied not only for the horizontal parallel subsurface drains, but also for parallel ditches or channels in the landscape or in another porous environment.

The relationship will be described between the subsurface drain parameters (drain spacing, drain depth and drain diameters), soil characteristics (hydraulic saturated conductivity, drainable pore space, also termed effective porosity, the thickness of the soil profile) and the drainage rate.

In the saturated flow conditions and according to the Dupuit's assumptions (Dupuit 1863) and Darcy's law (Darcy 1856), the flow velocity v (M.T^{-1}) in the horizontal x-direction can be expressed as:

$$v = -K\frac{\partial h}{\partial x} \qquad (1)$$

The change in water storage per unit surface area at an infinitely small period of time will be described by equation of continuity:

$$\frac{\partial (hv)}{\partial x} = -P\frac{\partial h}{\partial t} \qquad (2)$$

where:

K – hydraulic conductivity (M.T^{-1})
h – height of the water table level (M)
P – drainable pore space or effective porosity (-)
x – horizontal x-direction (x-coordinate) (M)
T – time unit

By substitution equation 1 to equation 2, the non-linear, partial differential equation of the second-order will be obtained:

$$\frac{\partial (hK[\partial h/\partial x])}{\partial x} = P\frac{\partial h}{\partial t} \qquad (3)$$

By the approximation h (M) = H (M) constant, in the first term of this equation, where H (M) = constant and represents the average depth of the aquifer, the equation 3 can be formed as:

$$HK\frac{\partial^2 h}{\partial x^2} = P\frac{\partial h}{\partial t} \tag{4}$$

This linearization applies mainly in case of deep impervious barriers and supposes, where H (M) converges to Hooghoudt's equivalent depth I' (M) and height of the water table above the level of drain can be neglected (Ritzema 2006).

In following case, the average depth of the aquifer H (M) was approached as

H (M) = I' + $\frac{h_0}{4}$ where h_0 (M) is an initial water table level (M) at time t = 0.

The equation 4 has the same shape as the Fourier's Equation for one-dimensional heat flow in orthogonal systems.

The equation 3 and 4 is also known as the Boussinesq's Equation (Boussinesq 1904). The unsteady-state saturated groundwater flow, without any recharges to the water table, is described just right by those equations.

The analytical solution of the equation 4 is based, besides others, on application of the Fourier's half-range sine series.

Dumm and Glover (Ritzema 2006, Wesseling 1969) for the lowering of an initially horizontal water table level h_0 (M), which shows Figure 1, shaped:

$$h(x,t) = \frac{4h_0}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}e^{-n^2 at}\sin(\frac{n\pi x}{L}) \tag{5}$$

For the next process will be used approximation with n = 1, with use of the first term of the equation 5 only, expressed as:

$$h(x,t) = \frac{4h_0}{\pi}e^{-a.t}\sin(\frac{\pi.x}{L}) \tag{6}$$

where:

h (x,t) – height of the water table (M) above the level of the drain at distance x (M) (from the drain pipe) at time t (T)

h_0 – initial water table level (M) at time t = 0

L – drain spacing (M)

t – time (T) after rise of the water table, (after period of recharge)

a – drainage intensity factor (T^{-1}), a = (π^2. K. H) / (L^2. P)

H – average depth of the aquifer (M), H (M) can be expressed as a function I', h_0, h(x,t)

I' – Hooghoudt's equivalent depth (M) of the soil layer below the level of the drain

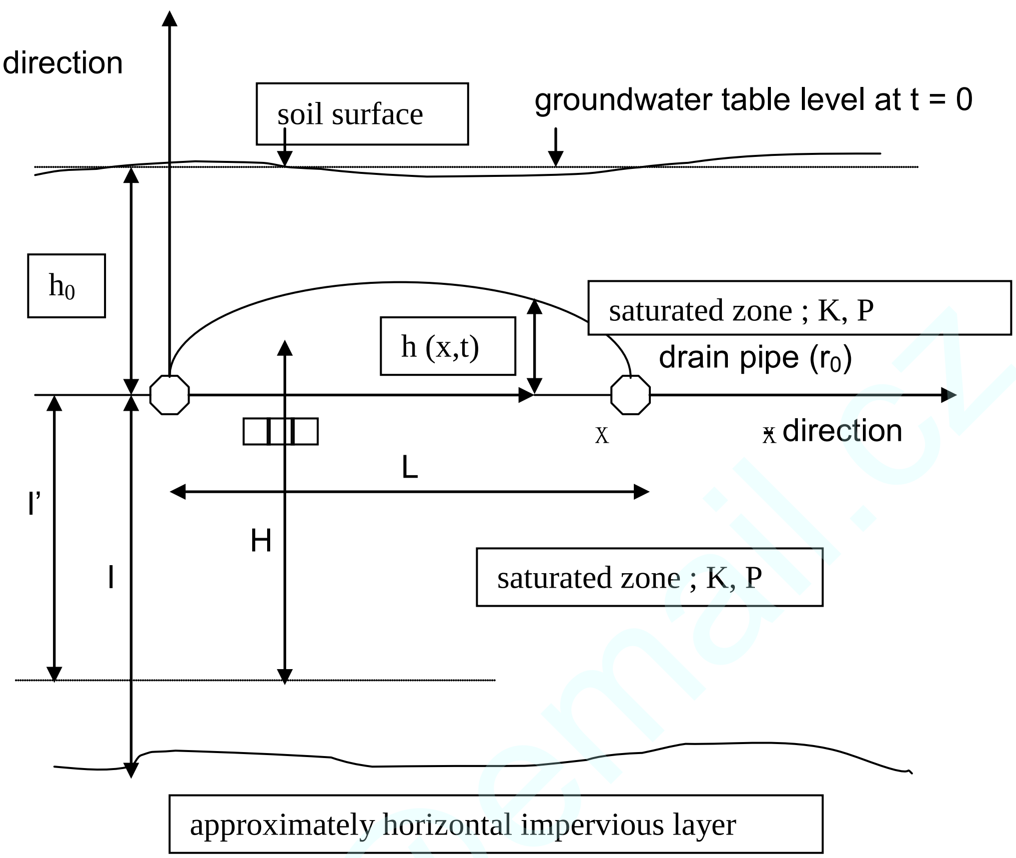

Figure 1. Height of the groundwater table h (x,t) at distance x > 0 at time t > 0 on saturated unsteady flow.

Stibinger (1985) showed the similar process of the analytical approximation of a linearized equation 4.
The parameter l' (M) is the function of the drain spacing L (M), of the drain diameters r_0 (M) and of the l (M) which is the real depth of the impervious floor below the level of the drain.
Approximation 5 and 6 describes relationship between the drain parameters [drain spacing L (M), drain depth h_d (M) and drain diameter r_0 (M)], soil characteristics [hydraulic conductivity K ($M.T^{-1}$), effective porosity P (-) and the thickness of the soil profile] and the lowering h(x,t) (M) of an initial water table level h_0 (M) as a function of time t (T) and place x-distance from the drain pipe (M) (see Figure 1).
Those analytical approximations are very well known methods, especially the application of the equation 6 for x = L / 2, where h(t) (M) is the height of the water table midway between the drains and can be calculated as:

$$h(x = L/2, t) = h(t) = \frac{4h_0}{\pi} e^{-at} \tag{7}$$

The use of the type of equation 7 is in a drainage engineering practice relatively frequent (Wesseling 1969, Dieleman 1976, Sagar α Preller 1980, Abdel-Dayem 1984, Ritzema 2006).
According Dieleman (1976) the equation 6 and 7 can be applied from the point of time, where the relation between drainage discharge rate and the lowering of the water table is constant (stable).
This point of time is defined as τ (days) = 0,4 (-) / a (days^{-1}).
Previous rough description of the saturated unsteady groundwater flow in the present subsurface parallel horizontal drainage system is by no means the main goal of this analysis.
Let it be only the starting point of the next sentences and thesis of the analytical approximation of the drainage retention capacity created by a subsurface pipe drainage system in the non-steady state conditions.

Analytical approximation drainage retention capacity R(t)

The amount of water above the drain, caused by floods, irrigation, spring's snowmelt or intensive rains is represented by h_0 (M) value.
After finishing of the floods, irrigation, spring's snowmelt or intensive rains processes, the initial water table level (at time t = 0) above the subsurface horizontal drainage pipes with drain spacing L (M) is h_0 (M).
The approximation of the shape of the initial water table level h_0 ($h(x,0)=h_0$, t=0 , $0 \le x \le L$) corresponds to the results of the water table observation from the drained part of the experimental watershed of RISWCPrague-Zbraslav and its groundwater regime.
These premises of the shape of the initial water table level h_0, expressed as $h(x,0)=h_0$ at t=0 for $0 \le x \le L$ harmonize with many authors as Dumm and Glover (Ritzema 2006, Wesseling 1969), Skaggs (1978), Sagar and Preller (1980), Shukla, Chauhan and Srivastava (1999), Upadhyaya and Chauhan (2001) and others.
The volume of the water above the two next parallel drains (at time t = 0) can be expressed as L . h_0 . P . 1 (see Figure 2) in the volume unites per unit of length, where 1 represents the unit of length.
It means, that at time t = 0, the area above the two next parallel drains, limited h_0 and L, multiplied P and multiplied 1, represents the volume of the water above the two next parallel drains in the volume unites, per unit of length [volume of water in M^3. M^{-1} is equal to L . h_0 . P . 1].
To get V(0) in the units of length per unit surface area, has to be expression [L.h_0.P.1] divided by [L.1]. This procedure can be described by equation in shape:

$$V(0) = \frac{L.h_0.P.1}{L.1} = h_0 P \tag{8}$$

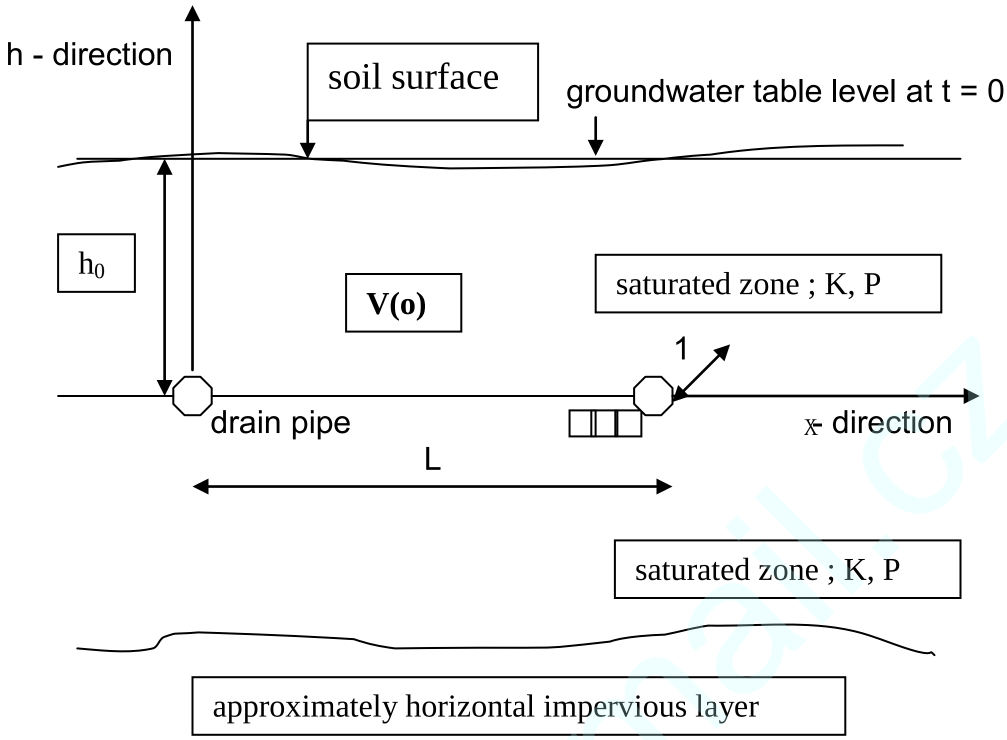

Figure 2. Mass of the water V(0) at time t = 0 above the drainage system level.

where V(0) (M) is volume (mass, quantity) of the water above the level of the parallel horizontal subsurface drainage pipe system (at time t = 0), expressed in the length units (M) per unit surface area.

All this process will be by the same way applied at time t > 0. The area above the two next parallel drains with drain spacing L (M) at the time t > 0 is approximately $\int_0^L h(x,t)dx$ (M^2) and by the same way as equation (8) can be formed as:

$$V(t) = [P.1.\int_0^L h(x,t)dx]/[L.1] = (P/L)\int_0^L h(x,t)dx \qquad (9)$$

where V(t) (M) represents the volume of water (water quantity) above the level of the parallel horizontal subsurface drainage pipe system (at time t > 0), expressed in the length units (M) per unit surface area (Figure 3).

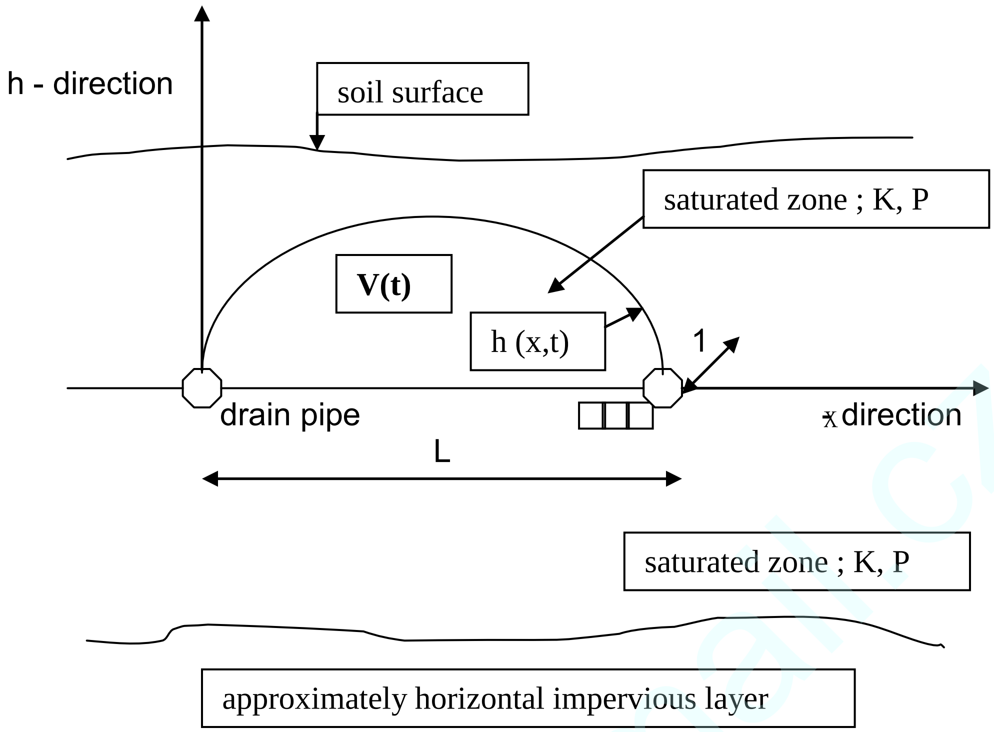

Figure 3. Mass of the water V(t) at time t > 0 above the drainage system level.

By substituting approximation 6 into the expression $\int_0^L h(x,t)dx$ and after integrating $(4.h_0/\pi)$. exp [- a.t] . $\int_0^L$ sin (π . x / L) dx , can be written:

$$\int_0^L h(x,t)dx = \frac{8h_0 L}{\pi^2} e^{-at} \tag{10}$$

By substituting equation 10 into equation 9, can be defined the formula for expression of V(t) (M). Equation 9 can be rewritten and shaped as:

$$V(t) = \frac{8h_0 P}{\pi} e^{-at} \tag{11}$$

The drainage retention capacity R(t) (M) at time t > 0 , expressed in the length units (M) per unit surface area, shown in Figure 4, can be described by this elementary balance relation as:

$$R(t) = V(0) - V(t) \qquad (12)$$

By substituting equation 8 and equation 11 into equation 12 and after rearrangement, the final form of the drainage retention capacity R(t) (M) can be defined as:

$$R(t) = h_0 P\left(1 - \frac{8}{\pi^2} e^{-at}\right) \qquad (13)$$

By this approximation, shaped in the equation 13, with knowledge of the basic subsurface drainage system parameters and soil hydrology characteristics (K, P, h_0) is possible to evaluate drainage retention capacity R(t) (M) in certain time t > 0 in the conditions where the initial water table level h_0 (M) at time t = 0 is identical with drain depth h_d (M). It means, that equation 13 can be formed as

$$R(t) = h_d P\left(1 - \frac{8}{\pi^2} e^{-at}\right) \qquad (14)$$

Drainage retention capacity R(t) (M) created by subsurface pipe drainage system and expressed in the length units (M) per unit surface area in a certain time t>0, is a groundwater reservoir limited by initially horizontal water table h_0 (M) at t=0 and intermediate position of water table with parabola shape above drain level at t>0.

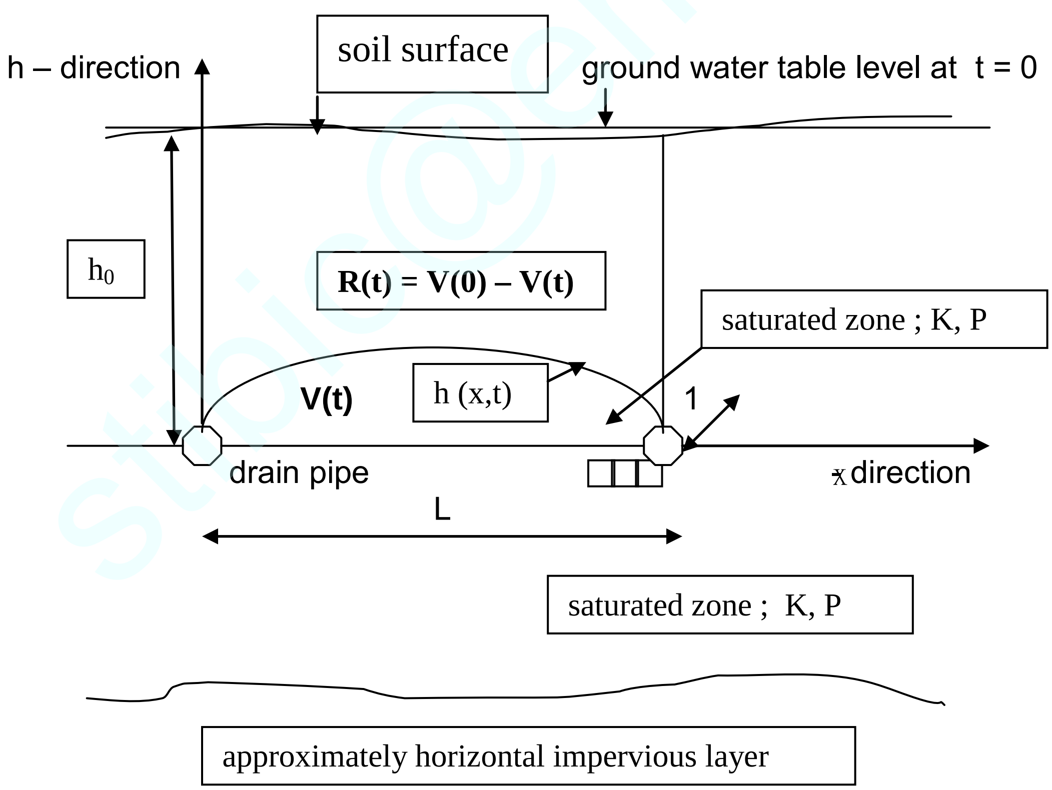

Figure 4. Drainage retention capacity R(t) at time t > 0.

If $h_d > h_0$ it is necessary to add to the value of the drainage retention capacity the value of groundwater reservoir between the initially horizontal water level h_0 (M) (t=0) and the soil surface, which is expressed in the length units (M) per unit surface area by formula $(h_d – h_0)$.P, then the retention capacity of soil layers $R(t)_1$ (M) can be formed as:

$$R(t)_1 = P(h_d - h_0) + h_0 P(1 - \frac{8}{\pi^2} e^{-at}) \tag{15}$$

After rearrangements equation 15 can be rearranged into:

$$R(t)_1 = P(h_d - h_0 \frac{8}{\pi^2} e^{-at}) \tag{16}$$

By substituting expression $h(t) = \frac{4h_0}{\pi} e^{-at}$ into the equation (16), the formula for direct calculating of drainage retention capacity $R(t)_1$ (M) by the values of the heights of the water table midway between the drains h(t) (M) can be formed as:

$$R(t)_1 = P\left[h_d - \frac{2}{\pi} h(t) \right] \tag{17}$$

References

Abdel-Dayem M. S. 1984. Drain spacing for falling water table in a clay cap over an artesian aquifer. In: Rycroft D. W. α Amer M. H. Prospect of the Drainage of Clay Soils (pp 45-46). Irrigation and Drainage Paper 51. FAO. Rome, Italy.

Boussinesq J. 1904. Recherches théoretiques sur l'écoulement des nappes d'eau infiltrées dans le sol et sur le débit des sources. Journal de mathématiques pures et appliquéss, no.10, part 5, pp. 5-78.

Darcy H. 1856. Les fontaines publique da la ville de Dijon. Dalmont, Paříž, Francie.

Dieleman P. J. and Trafford B. D. 1976. Drainage testing. Irrigation and Drainage Paper 28. FAO. Rome, Italy.

Dumm L. D. 1954. Drain spacing formula. Agricultural Engineering 35, pp. 726-730.

Dupuit J. 1863. Ethudes theoretiques et pratiques sur le mouvement des eaux dam les canaux découverts et a travers les terrains permeables. Dunod, Paris. France.

Ritzema H. P. 2006. Subsurface flows to drains. In: H. P. Ritzema (Ed) Drainage Principles and Applications (pp. 283-294). ILRI Publ. 16, Wageningen, The Netherlands.

Sagar B. and Preller Ch. 1980. Stochastic model of water table under drainage. Journal of the Irrigation and Drainage Division. ASCE Vol. 106 No. IR3:189-202.

Shukla K. N., Chauhan H. S. and Srivastava V. K. 1999. Transient drainage to partially penetrating drains in sloping aquifers. Journal of Irrigation and Drainage Engineering. Vol. 125, No. 5. ASCE, ISSN 0733-9437, pp. 246-253.

Skaggs R. W. 1978. Effect of drain tube openings on water-table drawdownd. Journal of the Irrigation and Drainage Division. Proceedings of the America Society of Civil Engineers, Vol. 127, No.IR1, pp. 13-21, New York, USA.

Stibinger J. 1985. Analytical solution of non-stationary drainage flow for systematic tile drainage and a single horizontal tile. Water Management, Series A, 39, No.5: 136-140. CTU Prague, Czech Republic. (In Czech, abstract in English).

Upadhyaya A. and Chauhan H. S. 2001. Falling water tables in horizontal / sloping aquifer. Journal of Irrigation and Drainage Engineering. Vol. 127, No. 6. ASCE, ISSN 0733-9437, pp. 378-384.

Wesseling J. 1969. Field drainage theories. In: 8th International Course on Land Drainage. Vol. 2.1 (pp 23-30). ILRI, Wageningen, The Netherlands.